NONSTEADY METHODS OF DETERMINING THE THERMOPHYSICAL CHARACTERISTICS OF BARRIER CONSTRUCTIONS

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The problem of determining the thermophysical characteristics of barrier (wall) constructions reduces to a problem in the theory of optimum control. For the solution of the latter we employ the Pontryagin optimization principle and we devise a computation scheme that is convenient for purposes of programming on an electronic digital computer. We present some calculational results based on experimental data derived in the thermophysical testing of the outside walls of various multipaneled buildings.

The thermophysical testing of wall designs is presently based entirely on the quantitative relationships governing the steady-state thermal regime [1]. However, the determination of thermal indices for outside walls on the basis of such tests involves a number of major drawbacks. First of all, the steady-state method makes it possible to determine only the thermal conductivities of the structural material; it is impossible with this method, however, to determine the heat capacity or the thermal diffusivity. Secondly, temperatures that are nonsteady are reduced by averaging to steady temperatures in the processing of the experimental results, thus making it impossible to achieve exact determinations of the required thermal indices. Thirdly, natural thermophysical tests can be carried out only under winter conditions, and the experiments last from 1 to 1.5 months in this case.

The familiar nonsteady methods of determining the thermal characteristics can be applied only to the study of specific specimens of structural materials, and only under laboratory conditions. These methods are exceedingly complex for full-scale structural models, and are entirely unsuited to testing under realistic operational conditions.

This paper develops a new approach to the problem, involving the use of the quantitative relationships

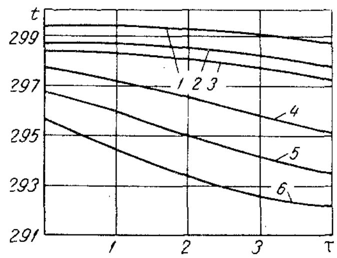

Fig. 1. Temperature curves in multilayer constructions (τ, hour, t, °K): 1) t_{in}, 2) t_1^*, 3) t_2^*, 4) t_3^*, 5) t_4^*, 6) t_{out}.

governing the nonsteady temperature field. We examine methods of determining the thermal conductivity and volume heat capacity of the material in individual layers of the construction; these methods are based on knowing the temperatures t_i^*, which vary during T at the boundaries of separation between these layers. These

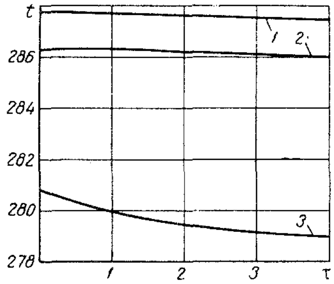

Fig. 2. Temperature curves in one-layer construction (τ, hour, t, °K): 1) t_{in}, 2) t_i^*, 3) t_{out}.

methods presume the use of a standard. The experimental work involves the following.

Thermoelectric sensors are mounted at a given level on the boundaries of separation between the layers and on the surfaces of the multilayer wall being investigated. A square plate (the standard)—with its center at that level—is attached tightly to the inside surface of the wall. An additional sensor is mounted at the center of the plate surface, and all of the thermoelectric sensors are connected to an automatic recording device. The standard plate must be fabricated of a homogeneous dry material whose thermophysical characteristics λ_0 and C_0 must be known in advance. The dimensions of this standard must ensure uniformity of the heat-transfer process at the section of the wall being studied, and the thickness Δx_0 must be commensurate with the thicknesses of the layers, which were assumed in the derivation of (5).

Thus, let us examine a multilayer construction consisting of the standard and n layers: Δx_0, Δx_1, Δx_2, ..., Δx_n and with λ_i and C_i that are constant for each of these layers. The uniform process of heat transfer in such a medium is described by the system [2]

$$C_i \frac{\partial t_i(\tau, x)}{\partial \tau} = \lambda_i \frac{\partial^2 t_i(\tau, x)}{\partial x^2}, \quad x_i \leqslant x \leqslant x_{i+1} \qquad (1)$$

under the conditions

$$t_i(0, x) = t_i^0(x),$$

$$t_0(\tau, 0) = t_{in}(\tau), \quad t_n(\tau, l) = t_{out}(\tau); \qquad (2)$$

$$t_{j-1}(\tau, x_j) = t_j(\tau, x_j),$$

Table 1

The Iteration Process of Determining the Thermophysical Characteristics of a Four-layer Construction

№	λ_1	λ_2	λ_3	λ_4	C_1	C_2	C_3	C_4	I
0	1.68000	0.07200	0.24000	1.68000	1680.000	168.000	1050.000	1680.000	15.58664
1	1.59600	0.07920	0.22800	1.59600	1848.000	193.600	1156.000	1848.000	5.29461
2	1.51200	0.08640	0.21600	1.51200	2016.000	201.600	1260.000	2016.000	0.65902
3	1.42800	0.09360	0.20400	1.42800	2100.000	218.400	1365.000	2100.000	0.16835
4	1.47000	0.09000	0.19200	1.47000	2016.000	210.000	1310.000	2016.000	0.11148
5	1.44900	0.08640	0.19800	1.44900	2058.000	214.200	1338.750	2058.000	0.09369
6	1.42800	0.08280	0.19500	1.45956	2037.000	212.100	1325.646	2037.000	0.02680
7	1.43850	0.07920	0.19656	1.45428	2016.000	214.150	1334.298	2047.500	0.00877
8	1.44900	0.07660	0.19572	1.44900	2026.500	214.200	1338.200	2058.000	0.00496
55	1.44300	0.05484	0.18276	1.44444	2090.634	115.526	1274.658	2096.724	0.00035
56	1.44168	0.05484	0.18278	1.44212	2091.978	115.752	1273.314	2098.026	0.00012
57	1.44240	0.05472	0.18278	1.44372	2097.522	115.872	1272.970	2099.328	0.00004

Table 2

The Iteration Process for the Determination of the Thermophysical Characteristics of a Sandwich Panel in a Building That Is in Use (the first method)

№	λ_1	λ_2	λ_3	λ_4	C_1	C_2	C_3	C_4	I
0	1.450	0.046	1.450	1.450	2006.40	43.89	2006.40	2006.40	15.88
1	1.268	0.058	1.268	1.268	2090.00	54.84	2090.00	2090.40	8.286
2	1.087	0.070	1.087	1.087	1839.20	65.79	1839.20	1839.20	3.567
3	0.906	0.081	0.906	0.906	1965.30	76.74	1965.30	1965.30	1.478
4	0.725	0.075	0.725	0.997	2090.00	71.27	2090.00	2090.00	0.640
5	0.815	0.081	0.544	1.087	2027.30	65.80	2027.30	2027.30	0.187
6	0.906	0.078	0.635	1.042	2058.65	60.33	2058.65	2058.65	0.049
24	1.049	0.071	1.015	1.014	2048.23	61.03	2084.57	2090.40	0.025
25	1.049	0.072	1.019	1.013	2062.99	60.94	2070.81	2084.98	0.023
26	1.050	0.074	1.024	1.012	2059.90	60.86	2077.04	2090.40	0.021

$$\lambda_{j-1}\frac{\partial t_{j-1}(\tau,\ x_j)}{\partial x}=\lambda_j\frac{\partial t_j(\tau,\ x_j)}{\partial x}. \qquad (3)$$

Here $0 \leq \tau \leq T$, $i = 0, 1, 2, \ldots, n$, $j = 1, 2, \ldots, n$. We impose the following constraints on λ_i and C_i:

$$0<\lambda_i^{\min}\leqslant\lambda_i\leqslant\lambda_i^{\max};\quad 0<C_i^{\min}\leqslant C_i\leqslant C_i^{\max}. \qquad (4)$$

The values of $\lambda_i^{\min}$, $C_i^{\min}$, $\lambda_i^{\max}$, and $C_i^{\max}$ are the limits within which the characteristics of the given type of material can vary. Assuming Δx_i to be quite small, we replace the right-hand members of (1) by finite 2-nd order differences [1, 3]. Then, for conditions (2) and (3) we have a system of ordinary differential equations

$$\frac{dt_i}{d\tau}=\frac{2}{C_{i-1}\Delta x_{i-1}+C_i\Delta x_i}\times$$

$$\times\left[\frac{t_{i-1}-t_i}{\Delta x_{i-1}}\lambda_{i-1}-\frac{t_i-t_{i+1}}{\Delta x_i}\lambda_i\right],$$

$$i=1,\ 2,\ \ldots,\ n, \qquad (5)$$

for the initial conditions

$$t_i(0)=t_i^0=\text{const}. \qquad (6)$$

Let us now present the mathematical formulation of the problem. We know the temperature curves $t_i^*(\tau)$ derived during the course of the experiment at the boundaries of separation between the layers of the construction within T hours under the condition of (2). We have to select those values of λ_i and C_i from (4) so that the solution of systems (5) and (6) for $t_i(\tau)$—derived for these values of λ_i and C_i with the specified degree of accuracy—coincides with $t_i^*(\tau)$, i.e., so that

$$I=\frac{1}{2}\sum_{i=1}^{n}\alpha_i\int_0^T[t_i(\tau)-t_i^*(\tau)]^2\,d\tau \qquad (7)$$

assumes a minimum value. The values of the standard characteristics λ_0 and C_0 are not included among the unknowns in this case.

This problem is an ordinary variational problem of optimum control. The minimization of the integral in (7) is reduced by familiar methods [4–6] to the determination of those values of λ_i and C_i which provide for the absolute maximum of the function

$$H=\sum_{i=1}^{n}\left[\frac{2}{C_{i-1}\Delta x_{i-1}+C_i\Delta x_i}\left(\frac{t_{i-1}-t_i}{\Delta x_{i-1}}\lambda_{i-1}-\right.\right.$$

$$\left.\left.-\frac{t_i-t_{i+1}}{\Delta x_i}\lambda_i\right)p_i-\alpha_i\frac{1}{2}(t_i-t_i^*)^2\right], \qquad (8)$$

where

$$\frac{\partial p_i}{\partial\tau}=-\frac{\partial H}{\partial t_i},\quad p_i(T)=0,\quad i=1,\ 2,\ \ldots,\ n. \qquad (9)$$

The maximization of the function H can be achieved by the gradient method [6]. The successive approximations in this case are chosen from the formulas

$$(\lambda_i)^{k+1}=\begin{cases}\lambda_i^{\min} & \text{when } \lambda_i^{\min}>(\lambda_i)^k+\Delta(\lambda_i)^k,\\ (\lambda_i)^k+\Delta(\lambda_i)^k & \\ \quad\text{when } \lambda_i^{\min}\leqslant(\lambda_i)^k+\Delta(\lambda_i)^k\leqslant\lambda_i^{\max}, & \\ \lambda_i^{\max} & \text{when } \lambda_i^{\max}<(\lambda_i)^k+\Delta(\lambda_i)^k,\end{cases} \qquad (10)$$

where

$$\Delta(\lambda_i)^k=\varepsilon_i^k\,\text{sign}\int_0^T\frac{\partial H}{\partial\lambda_i}d\tau,\quad \varepsilon_i^k>0,$$

$$i=1,\ 2,\ \ldots,\ n;\quad k=0,\ 1,\ 2,\ \ldots. \qquad (11)$$

These same formulas apply also to C_i.

The programming procedure involves the solution of the system of differential equations (5), (6), and (9); in addition, it involves the determination of the unknowns from formulas (10) and (11). The initial values of λ_i and C_i (when $k = 0$) are chosen from conditions (4). If $\int_0^T\frac{\partial H}{\partial\lambda_i}d\tau$ change sign in the next step of the approximation in this case, the corresponding ε_i^k are set equal to $\varepsilon_i^k/2$. However, if the quantities $\int_0^T\frac{\partial H}{\partial\lambda_i}d\tau$ retain their signs, the values of ε_i^k are retained, but with a slowing down of the convergence process it is advisable to set these equal to $2\varepsilon_i^k$. Initially the values of ε_i^0 are chosen arbitrarily but $\varepsilon_i^0 < \lambda_i$. This process is continued until the required degree of smallness for I is achieved.

Let us now assume that the curves for the temperature t_i^* are not known for all of the separation boundaries x_i which were assumed in the derivation of (5). This may be the case if the recording t_i^* at this particular boundary during the course of the experiment was omitted. In a number of cases involving the study of multilayer walls under natural conditions this is a result of the difficulty involved in the mounting of the thermoelectric sensors at the proper boundaries of separation, while in the case of uniform constructions this operation is unnecessary. On the other hand, the thicknesses Δx_i of the layers may prove to be somewhat too large for the approximation of (1) by system (5), and to raise the accuracy it becomes necessary to introduce additional scale markings into the segment $[0, l]$. In each case, the values of λ_i for the layers at whose boundaries t_i^* is unknown must be assumed equal to each other (the same applies to C_i). In the case of multilayer constructions these quantities denote the reduced values of λ_i and C_i of the corresponding layers. This scheme for the solution of the problem does not change in these cases, if we assume the appropriate $\alpha_i = 0$ in (7). However, from the practical standpoint, it is convenient to distinguish two methods in this case: 1) the determination of the characteristics for the individual layers of a multilayer construction; and 2) the determination of the characteristics for a uniform construction (in the case of a multilayer construction, we have reference to the reduced values of the characteristics). Both of the methods provide for the use of a standard layer. In the first case, the sensors must be mounted at the boundaries

Table 3

The Iteration Process for the Determination of the Thermophysical Characteristics of a Sandwich Panel in a Building That is in Use (the Second Method)

№	λ	C	I
0	0.1392	1387.76	0.5633
1	0.1740	1214.29	0.3561
2	0.2088	1040.82	0.1592
3	0.2436	867.35	0.0266
4	0.2784	693.88	0.1378
5	0.2610	780.62	0.0317
6	0.2436	867.35	0.0266
47	0.2025	671.31	0.0016
48	0.2032	670.14	0.0015
49	0.2039	668.8	0.0015

of separation between the layers, while in the second case there is no need for such an operation.

We note that if t_i^* is unknown, the corresponding initial conditions in (6) are also unknown. In the solution of (5) these conditions must be restored by interpolation of the quantities known from (6), which leads to an error in the solution of (5). The magnitude of this error is the smaller, the closer the temperature is to the steady state. In such cases the thermal-engineering tests must therefore be started after sufficiently stable temperatures have been established within the wall. On the other hand, the magnitude of the error in the interpolation can be reduced by the solution of (4) during time $T' > T$. With sufficiently large T', the effect of the conditions in (6) at the instant $\tau = T' - T$ is small, and the solutions of (5) during the time $T' - T$ can be omitted.

We will illustrate this method by means of several examples, most of which are the results of natural thermophysical tests of wall constructions. A "Ural-4" computer was used for the solution of these problems.

1. We examine the temperature curves for $t_1^*(\tau)$, $t_2^*(\tau)$, and $t_3^*(\tau)$, derived for a 4-layer construction by calculation under the following conditions: $l = 0.31$; $T = 7200$ sec; $\Delta x_0 = 0.05$; $\Delta x_1 = 0.04$; $\Delta x_2 = 0.05$; $\Delta x_3 = 0.075$; $\Delta x_4 = 0.095$; $\lambda_0 = 0.058$; $C_0 = 277.97$; $\lambda_1 = 1.440$; $C_1 = 2100.00$; $\lambda_2 = 0.054$; $C_2 = 113.40$; $\lambda_3 = 0.180$; $C_3 = 1260.00$; $\lambda_4 = 1.440$; $C_4 = 2100.00$; $t_i(0, x) = 293.16$; $t_{in} = t_{out} = 273.16$. We are confronted with the problem of restoring the values of λ_i and C_i ($i = 1, 2, 3, 4$) when these are arbitrarily chosen (for example, for the values of λ_i and C_i in the zero line of Table 1). This problem is presented here to illustrate the nature of the convergence in the method being employed. The errors in the results of the 57-th approximation with respect to the above-cited values, on the average, do not exceed 1.7%.

Here and in the following cases, the solution of Eqs. (5), (6), and (9) is achieved by the Runge-Kutta method, with a constant interval $h = 112.5$ sec. However, the output of the results and the calculation of the various integrals involve an interval of $h = 900$ sec. In this case, eight ($n = 8$) of the equations in (5) are solved everywhere, i.e., additional scale markings are introduced into the layers under consideration to increase the accuracy of the solution of (5), but without affecting the final results. For example, in this problem each of the layers Δx_i is divided into two equal parts and in (7) $\alpha_1 = \alpha_3 = \alpha_5 = \alpha_7 = 1$, $\alpha_2 = \alpha_4 = \alpha_6 = \alpha_8 = 0$.

2. We are investigating the sandwich wall panel of a building that is in use, said panel consisting of an inside reinforced-concrete layer $\Delta x = 0.035$, a slag-cotton plate $\Delta x = 0.05$, and an outside reinforced-concrete layer $\Delta x = 0.075$, divided into two parts with thicknesses of 0.04 and 0.035. As the standard layer we have used a PVC plate having the dimensions 0.9×1.2, $\Delta x_0 = 0.06$, $\lambda_0 = 0.0626$, and $C_0 = 247.5$. The experiments were carried out to apply both of the above-described methods, and the temperature graphs for $t_{in}(\tau)$, $t_1^*(\tau)$, $t_2^*(\tau)$, $t_3^*(\tau)$, $t_4^*(\tau)$, and $t_{out}(\tau)$ are given in Fig. 1. The layers were further divided in the solution of the problem, and the inadequate initial conditions in (6) were reinforced by the linear interpolation of the values of t_i^* for the case in which $\tau = 0$. The results of the calculations are presented in Table 2. The formal definition of the thermal resistance of the wall as $R = \sum_{i=1}^{n} \frac{\Delta x_i}{\lambda_i}$ in this case yields $R = 0.793$.

In applying the second method, we have used only the temperatures $t_{in}(\tau)$, $t_1^*(\tau)$, and $t_{out}(\tau)$ (Fig. 1). The calculation results are presented in Table 3. The value of the thermal resistance in this case is $R = 0.784$.

3. The thermophysical studies were carried out on the facing panel used in the "P" version of the II-49

Table 4

The Iteration Process for the Determination of the Thermophysical Characteristics of an Outside Panel in a Building of the P-49 "II" Series

№	λ	C	I
0	0.4640	1003.20	0.0031
1	0.4060	1254.00	0.0061
2	0.4350	1128.60	0.0014
3	0.4640	1003.20	0.0031
4	0.4489	1065.90	0.0013
5	0.4350	1128.60	0.0014
6	0.4420	1097.25	0.0011
13	0.4222	1084.46	0.0012
14	0.4211	1089.39	0.0011
15	0.4199	1093.32	0.0011

building series; the facing panel was made up of a 'keramzit'-concrete combination, in which the 'keramzit' is a porous clay filler for cement. The panel exhibited a thickness of 0.4. Since the textured layers are of insignificant thickness, the construction is assumed to be uniform and the calculations were carried out only with application of the second method. However, the initial conditions were established by means of measurement. For the standard layer we employed a polyurethane-foam plate $\Delta x_0 = 0.05$, $\lambda_0 = 0.074$, and $C_0 = 178.50$. The construction being investigated here was divided into eight arbitrary layers of identical thickness. The temperature graph is shown in Fig. 2, and the calculation results are given in Table 4.

Our experience in the use of this method has demonstrated that with the proposed procedures it becomes possible, within a short period of time, to determine all of the thermophysical characteristics of a barrier [wall] construction.

NOTATION

τ is the time, sec; x is the coordinate, m; x_i is the separation-point coordinate of the (i − 1)-th and i-th beds; l is the total thickness of the construction; Δx_i is the thickness of the i-th bed of the construction, m; $t_i(\tau, x)$ is the temperature of the i-th bed, °K; t_i is the temperature at the boundary of the (i − 1)-th and i-th beds, calculated; t_i^* is the temperature at the boundary of the (i − 1)-th and i-th beds known from experiment; $t_i^0(x)$ is the initial temperature of the i-th bed; t_i^0 is the initial temperature at the boundary of the (i − 1)-th and i-th beds; $t_{in}(\tau)$ and $t_{out}(\tau)$ is the temperatures of inside and outside surrounding surfaces; λ_i is the thermal conductivity of the i-th bed, W/m · deg; C_i is the volumetric heat capacity of the i-th bed, kJ · kg/ /deg · m^2; R is the thermal resistance, m^2 · deg/W.

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